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**CS-556-B**

**Solutions**

**Ans1:**

**Ans2:** Given,

Two vectors .

The span is is the set of linear combinations of the vectors  and .

In other terms, span

So, Span = all linear combinations of and .

As, the (linear) span is a vector space. By definition it is the smallest vector space that contains all the elements in the set. In particular it includes all linear combinations of those elements (and will in fact contain exactly all linear combinations that can be formed with those elements).

Hence, the two vectors and forms the standard basis of the vector space .

Therefore, they span and so any two linearly dependent vectors in .

**Ans3:** Given,

Dot Product of vectors and :

**Ans4:** Given,

Using Dot Product formula,

**Ans5:** Given,

If has the only solution , then are linearly independent.

Forming a matrix and row reducing it,

Hence, only .

Therefore, are linearly independent vectors.

**Ans6:**

**Part 1:** Suppose there is a subset of a vector space is linearly independent. We need to prove that cannot be expressed as a linear combination of elements of with non-zero coefficients.

By contradiction, assume that

Where are elements of S and not all coefficients are equal to 0. Let be a non-zero coefficient.

By subtracting from both sides and dividing the resulting equality by we get:

where is missing in the left-hand sum. This equality implies that is a linear combination of other elements of . We obtained a contradiction with our assumption that is linearly independent, which completes the proof.

**Part 2:** Now we need to prove that if cannot be expressed as a linear combination of elements of with non-zero coefficients then is linearly independent. By contradiction, suppose that is not linearly independent. Then there exists an element in which is equal to a linear combination of other elements of :

By subtracting s from both sides, we get:

Thus, we have a linear combination of distinct elements of which is equal to and not all coefficients of this linear combination are equal to zero (for example, the coefficient of is -1).

**Hence Proved.**

**Ans7:** Given,

Demonstrate:

LHS:

RHS:

As LHS = RHS,

**Hence Proved.**

**Ans8:** Given,

Calculating Minors,

Calculating Determinant,

Calculating Inverse of Matrix A,

**Ans9:** Given,

Calculating Minors,

Calculating Determinant,

Calculating Inverse of Matrix A,

Representing changed basis for defined matrix A,